UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE
MATHEMATICS

COURSE CODE: **MAA 123**

COURSE TITLE: CALCULUS II

DATE: 21/4/2023 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a. Define the terms
   i. Definite integral
   ii. Indefinite integral (3 marks)

b. State the Mean Value Theorem for integrals (2 marks)

c. Use fundamental theorem to evaluate (3 marks)

   i. $\frac{d}{dx}\int_0^x \frac{1}{1+t^2}dt$

   ii. $\int_0^{\pi} \cos x\, dx$

d. Evaluate

   i. $\int \frac{6x+7}{(x+2)^2}dx$ (4marks)

   ii. $\int \frac{2x^3-4x^2-x-3}{x^2-2x-3}dx$ (6 marks)

   iii. $\int_5^6 \frac{2x^2-13x+13}{(x-4)^3}dx$ (5 marks)

e. The area enclosed by the curve $y = 2e^{x/2}$, the x-axis and ordinates x = -2 and x = 3 is rotated about the x-axis. Calculate the volume generated. (4 marks)

f. Evaluate $\int x^2 e^x dx$ (3 marks)

QUESTION TWO (20 MARKS)

a. Find the average value of $f(x) = 3 - \frac{3x}{2}$ on the interval $[0,2]$ and the point at which f takes this value in the domain. (7 marks)

b. Evaluate $\int e^{ax} \sin bx \ \ dx$ (7 marks)

c. Sketch the curve and find the area enclosed by the curve y= sin 2x, the x-axis and the given ordinates $\left(0, \frac{\pi}{2}\right)$ (6 marks)

QUESTION THREE (20 MARKS)

a. Evaluate $\int \frac{x-8}{x^2-x-2}dx$ (6 marks)

b. Show that if f is continuous on $[a,b]$, then $F(x) = \int_a^x f(t)dt$ is continuous on $[a,b]$ and differentiable on (a,b) and its derivative is $f(x)$: $\frac{d}{dx}F(x) = \int_a^x f(t)dt = f(x)$ (6 marks)

c. Find the area bounded by the curves $y = 4 - x^2$ *and* $y = x^2 - 2x$ (5 marks)

d. Evaluate $\int x \sin x dx$ (3 marks)

QUESTION FOUR (20 MARKS)

a. Find the volume of a solid generated by revolving the region bounded by $y = \frac{1}{x^2}$ and the lines $x = 2$ and $y = 4$ about the y-axis (6 marks)

b. Evaluate $\int Cos^5 x Sin x dx$ (6 marks)

c. Evaluate $\int_1^2 3e^{4x}dx$ (3 marks)

d. Evaluate $\int x^2 \ln x dx$ (5 marks)

QUESTION FIVE (20 MARKS)

a. Compute $\int Sin^4 x dx$ (9 marks)

b. Evaluate the following integrals

i. $\int Cos^5 x\, dx$ (6 marks)

ii. $\int Cos^2(3x)dx$ (5marks)