# UNIVERSITY EXAMINATIONS

## 2022/2023 ACADEMIC YEAR

## SECOND/THIRD YEAR SECOND SEMESTER

## MAIN EXAMINATION

## FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS AND BACHELOR OF SCIENCE PHYSICS

**COURSE CODE** MAA 222/MAT322

**COURSE TITLE** **OPERATIONS RESEARCH**

**DATE: 26/4/2023 TIME: 09:00 A.M – 11:00 A.M**

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**INSTRUCTIONS TO CANDIDATES**

Answer Question ONE and any other TWO questions.

## QUESTION ONE (30 MARKS)

a) Explain the following terms as used in operation research
   i) Surplus variable (1 mark)
   ii) Basic variable (1 mark)
   iii) Transportation problem (1 mark)
   iv) Post optimality (1 mark)

b) Describe the types of transportation problem. (3 marks)

c) Consider the following table containing figures that relates to contribution from service engineer and that it is required to maximize contribution (use hungarian method). (8 marks)

| | W | X | Y | Z |
|---|---|---|---|---|
| 1 | 25 | 18 | 23 | 14 |
| 2 | 38 | 15 | 53 | 23 |
| 3 | 15 | 17 | 41 | 30 |
| 4 | 26 | 28 | 36 | 29 |

d) Use simplex method to
   Max $z = 2x_1 + 5x_2$
   Subject to
   $x_1 + x_2 \leq 10$
   $2x_1 + x_2 \leq 8$
   $x_1, x_2 \geq 0$ (9 marks)

e) Use graphical method to obtain the optimum solution to the linear programming problem.
   Max $z = 5x + 4y$
   Subject t
   $6x + 4y \leq 24$
   $x + 2y \leq 6$
   $-x + y \leq 1$
   $y \leq 2$
   $x, y \geq 0$ (6 marks)

## QUESTION TWO

a) Write a linear programming model for the General transportation problem (3 marks)

b) Form the dual of the following LP problem

Max $z = 3x_1 + 4x_2 + 5x_3$

Subject to

$6x_1 + 7x_2 + 8x_3 \leq 25$

$7x_1 + 2x_2 + 9x_2 \leq 40$

$x_1, x_2, x_3 \geq 0$ (4 marks)

c) Consider the following LP problem

Max $z = 6x_1 + 8x_2$

Subject to

$5x_1 + 10x_2 \leq 60$

$4x_1 + 4x_2 \leq 40$

$x_1, x_2 \geq 0$ (8 marks)

i) Solve the LP

ii) Hence find the new solution if

I) The right hand side constraints of the constraint 1 and constraint 2 are changed from 60 to 40 and 40 to 20. (3 marks)

II) If a new constraint $7x_1 + 2x_2 \leq 65$ is added (2 marks)

## QUESTION THREE (20 MARKS)

a) Explain the following terms (2 marks)

i) Assignment problem (2 marks)

ii) Transshipment problem

b) Obtain the initial basic solution using the following methods in the problem below. (5 marks)

i) North west corner cell method (5 marks)

ii) Least cost cell method (6 marks)

iii) Vogel's approximation method

| | D | E | F | G | Available (Supply) |
|---|---|---|---|---|---|
| A | 11 | 13 | 17 | 14 | 250 |
| B | 16 | 18 | 14 | 10 | 300 |
| C | 21 | 24 | 13 | 10 | 400 |
| Requirement (Demand) | 200 | 225 | 275 | 250 | 950 |

**QUESTION FOUR (20 MARKS)**

a) Solve the following problem using Big M method. (13 marks)

Min $Z = 10x_1 + 15x_2 + 20x_3$

Subject to

$$2x_1 + 4x_2 + 6x_3 \geq 24$$

$$3x_1 + 9x_2 + 6x_3 \geq 30$$

$$x_1, x_2, x_3 \geq 0$$

b) A company has three factories located in three cities X, Y, Z. This factory supplies consignments to fur dealers A, B, C and D. The dealers are spread all over the country. The production capacity of these factories is 1000, 700 and 900 units per month respectively. The net return by unit is given in the table below.

| Factories | A | B | C | D | Capacities |
|---|---|---|---|---|---|
| X | 6 | 6 | 6 | 4 | 1000 |
| Y | 4 | 2 | 4 | 5 | 700 |
| Z | 5 | 6 | 7 | 8 | 900 |
| Requirement | 900 | 800 | 500 | 400 | 2600 |

Obtain basic feasible solution using North West corner method. (7 marks)

**QUESTION FIVE (20 MARKS)**

A company wants to produce three products A, B and C. The unit profits of these products are K£ 4, K£ 6 and K£ 2 respectively. These products require two types of resources - man power and material. In the following L.P model is formulated for determining the optimal product mix.

Max $z = 4x_1 + 6x_2 + 2x_3$

Subject to

$x_1 + x_2 + x_3 \leq 3$ (man-power)

$x_1 + 4x_2 + 7x_3 \leq 9$ ( material )

Where $x_1$, $x_2$, $x_3$ are the number of products A, B, and C produced.

a) Find the optimal product mix and the corresponding profit to the company (6 marks)

b) What happens if $C_3$ is increased to K£ 12? What is the new optimal product mix in this case? (4 marks)

c) i) Find the range on basic coefficient $C_1$ such that the current optimal product mix remains optimal. (4 marks)

ii) Find the effect when $C_1$ = K£ 8 on the optimal product mix. (6 marks)