# UNIVERSITY EXAMINATIONS

## 2022/2023 ACADEMIC YEAR

## THIRD YEAR SECOND SEMESTER

## MAIN EXAMINATION

## FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

**COURSE CODE:** **MAA 326**

COURSE TITLE: **ODE II**

**DATE:** 19/04/2023 **TIME:** 9 AM -11 AM

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## INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

**QUESTION ONE (30 MARKS)**

(a) (i) Define the Gamma function (2 mks)

(ii) Show that $\Gamma(x-1) = (x-2)\Gamma(x-2)$ (4 mks)

(b) (i) Find the general solution of a system of differential equation.

$$X^{I} = \begin{pmatrix} -4 & 1 & 1 \\ 1 & 5 & -1 \\ 0 & 1 & -3 \end{pmatrix} X$$ (8 mks)

(ii) Show that the solutions in (i) above are linearly independent (3 mks)

(c) Use elimination method to solve the system (8 mks)

$$Dx + (D+2)y = 1$$

$$(D-3)x - 2y = t$$

(d) Compute $e^{At}$ given that $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (5 mks)

**QUESTION TWO (20 MARKS)**

(a) Solve $X^{I} = AX + B(t)$ for

$A = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}$ ; $B(t) = \begin{bmatrix} 12t - 11 \\ -3 \end{bmatrix}$ (10 mks)

(b) Apply Picard's method to solve the following initial value problem up to 3rd approximation

$\frac{dy}{dx} = x - 3y + 4$: $x = 0, y = 3$ (10 mks)

## QUESTION THREE (20 MARKS)

(a) Solve the differential equation given that $y = e^x$ is a solution

$$x\frac{d^2y}{dx^2} - (x+1)\frac{dy}{dx} + y = 0$$

(10 Marks)

(b) Consider the system

$$\frac{dx_1}{dt} = -x_1 + x_2$$

$$\frac{dx_2}{dt} = -x_2 + 2x_1 - x_1x_3$$

$$\frac{dx_3}{dt} = -x_3 + x_1x_2$$

Linearize the system

(10 mks)

## QUESTION FOUR (20 MARKS)

(a) Define the term Bifurcation

(2 mks)

(b) Difine the following critical points, in each case draw the phase portrait

(i) Spiral (3 mks)

(ii) Centre (3 mks)

(iii) Node (3 mks)

(c) Verify that on the interval $(-\infty, \infty)$ $X_1 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} e^t$ and $X_2 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} e^{3t}$ and

$X_3 = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} e^{5t}$ are fundamental solutions of $X' = \begin{pmatrix} 3 & -2 & 0 \\ -1 & 3 & -2 \\ 0 & -1 & 3 \end{pmatrix} X$ (9 mks)

**QUESTION FIVE (20 MARKS)**

(a) What is the existence of uniqueness theorem of a system of differential equation? (2 mks)

(b) Replace the equation by a system of first order

$y^{III} + Py^{II} - Qy^{I} - Ry = 10x$ (4 mks)

(c) Consider the system of differential equations

$$\frac{dx}{dt} = x - 2y + 2z$$

$$\frac{dy}{dt} = -2x + y - 2z$$

$$\frac{dz}{dt} = 2x + z - 2y$$

(i) Find the general solution to the system (11 mks)

(ii) Find the particular solution given the initial value $x(0) = 0$, $y(-3) = 0, z(0) = 0$ (3 mks)