UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FORTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAA 412/MAT 421**

COURSE TITLE: **PARTIAL DIFFERENTIAL EQUATION I**

DATE: 19/04/2023 **TIME: 2:00 PM – 4:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Obtain a first order partial differential equation from $z = x^2 + y^2 + (z-a)^2 = b^2$, a and b being constants. (4 marks)

b) Show that the surfaces

$$F(x,y,z) = x^2 + 4y^2 - 4z^2 - 4 = 0$$
$$G(x,y,z) = x^2 + y^2 + z^2 - 6x - 6y + 2z + 10 = 0$$

Are tangent at the point (2,1,1). (6 marks)

c) By direct integration solve $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$; $z(x,0) = x^2$, $z(1,y) = Cosy$ (6 marks)

d) Using the method of multipliers, solve:

$$(mz - ny)p + (nx - lz)q - ly + mx = 0.$$ (8 marks)

e) Solve the Non-linear Partial Differential equation by first expressing it in standard form; $x + y = \frac{p^2+q^2}{z^2}$ (7 marks)

QUESTION TWO (20 MARKS)

a) Consider the Linear Equation of the type $\mathbf{Pp} + \mathbf{Qq} = \mathbf{R}$, where $\boldsymbol{P}$, $\boldsymbol{Q}$ and $\mathbf{R}$ are functions of x, y, z and $\mathbf{p}$ and $\mathbf{q}$ are differential co-efficients. Show that $\mathrm{f(u,v)} = 0$ where u, v are functions of x, y, z is the required solution. (12 marks)

b) By direct integration, solve $\frac{\partial^2 z}{\partial y^2} = z$; $y = 0$ then $z = e^x$, $\frac{\partial z}{\partial y} = e^{-x}$ (8 marks)

QUESTION THREE (20 MARKS)

a) Use Charpit's method to find the complete integrals of the differential equation $(p^2 + q^2)y = qz$

(10 marks)

b) Use Jacobi's method to find a complete integral of the equation $p^2 x + q^2 y = z$. (10 marks)

QUESTION FOUR (20 MARKS)

a) Solve $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0.$ (12 marks)

b) By eliminating arbitrary functions, obtain the partial differential equation from: $z = f(x+ct) + g(x-ct)$. (8 marks)

QUESTION FIVE (20 MARKS)

a) Find the general solution of the Lagrange equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)z$. (10 marks)

b) Using an appropriate method, solve the Partial Differential Equation $2(z + xp + yq) = yp^2$. (10 marks)