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**UNIVERSITY EXAMINATIONS  
2022/2023 ACADEMIC YEAR  
FIRST YEAR SECOND SEMESTER  
MAIN EXAMINATION  
FOR THE DEGREE OF BACHELOR OF SCIENCE  
MATHEMATICS**

**COURSE CODE: MAP 121**

**COURSE TITLE: ALGEBRAIC STRUCTURES I**

**DATE: 25/4/2023**

**TIME: 2:00 PM – 4:00 PM**

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

**QUESTION ONE COMPULSORY (30 MARKS)**

- a) Define the following
- i. Binary operation (2marks)
  - ii. Group (4marks)
  - iii. Composition of functions (2marks)
- b) Prove that the identity element of a group is always unique (4marks)
- c) Describe the Dihedral group  $D_5$  (8marks)
- d) Let  $G$  be a group and  $H \trianglelefteq G$  prove that the the set  $G/H$  forms a group (6marks)
- e) State four examples of groups (4marks)

**QUESTION TWO (20 MARKS)**

- a) Define the following
- i. Subgroup (3marks)
  - ii. Klein-four group (2marks)
  - iii. Alternating group (2marks)
- b) State the properties of a ring (3marks)
- c) State the distinct left cosets of  $\langle 4 \rangle$  in  $\mathbb{Z}_{12}$  (5marks)
- d) Prove that a finite group  $G$  whose order is a prime  $p$  is cyclic (5marks)

**QUESTION THREE (20 MARKS)**

- a) Define the following
- i. Ring (5marks)
  - ii. Normal subgroup (2marks)
  - iii. Quaternion group (2marks)
- b) Given the set  $S_3 \cong \langle (13) \rangle$ , state the distinct cosets of  $\langle (13) \rangle$  in  $S_3$  (5marks)
- c) Prove that every subgroup of an abelian group is normal (4marks)
- d) State the properties of a Field (2marks)

**QUESTION FOUR (20 MARKS)**

- a) Find the inverse of the following matrix, whose entries are elements of  $Z_6$  (6marks)

$$A = \begin{bmatrix} 1 & 2 \\ 5 & 5 \end{bmatrix}$$

- b) State the Lagrange's theorem (2marks)  
c) Construct the Cayley table for the Klein-4 group and prove its abelian (8marks)  
d) State 4 examples of binary operations (4marks)

**QUESTION FIVE (20 MARKS)**

- a) Define the following terms
- i. Abelian group (2marks)
  - ii. Proper subgroup (2marks)
  - iii. Cyclic group (2marks)
  - iv. Range of a function (2marks)
- b) Discuss the symmetric group  $S_3$  and give its subgroups and prove if it satisfies the Lagrange's theorem (12marks)