UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

SECOND YEAR

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: MAP 221

COURSE TITLE: LINEAR ALGEBRA II

DATE: 10/8/2023 **TIME**: 8:00 AM – 10:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Find general quadratic form of

$$E = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$ (5 Marks)

b) Find the characteristic equations of

$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Are they similar? Explain (10 Marks)

c) Find the matrix representation of the linear map $F: IR^3 \to IR^2$ defined by $F(x,y,z) = (2x + 3y - z, 4x - y + 2z)$ relative to the basis of IR^3 and IR^2 respectively

$B_1 = \{\mu_1 = (1,1,0), \mu_2 = (1,2,3), \mu_3 = (1,3,5)\}$

$B_2 = \{v_1 = (1,2), v_2 = (2,3)\}$ (15 Marks)

Question Two (20 Marks)

a) Find the eigenspaces of the matrix (12 Marks)

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

Question Three (20 Marks)

Let $S = \{\mu_1 = (1,2,0)\ \mu_2 = (1,3,2)$ and $\mu_3 = (0,1,3)\}$

$S' = \{v_1 = (1,2,1)\ v_2 = (0,1,2)\ v_3 = (1,4,6)\}$

i) Find the change of basis matrix P from S to S^1 (8 Marks)

ii) Find the change of basis matrix Q from S^1 bank to S (8 Marks)

iii) Verify that $Q = P^{-1}$ (4 Marks)

Question Four (20 Marks)

a) Find the matrix representation of the linear mapping $F: IR^3 \to IR^2$ defined by $F(v) = Av$ relative to the bases

$B_1 = \{\mu_1 = (1,1,1)\ \mu_2 = (1,1,0)\ \mu_3 = (1,0,0)\}$

$B_2 = \{v_1 = (1,3)\ v_2 = (2,5)\}$

$$A = \begin{pmatrix} 2 & 5 & -3 \\ 1 & -4 & 7 \end{pmatrix}$$ (12 Marks)

b) Find the matrix representation of the linear operator $S: IR^2$ defined by $S(x,y) = (2y, 3x - y)$ relative to the basis

$B = \{v_1 = (1,3)\ v_2 = (2,5)\}$ (8 Marks)

Question five (20 Marks)

a) Find the general quadratic form of

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$ (4 Marks)

b) Show that $A = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 9 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ is positive (6 Marks)

c) If $u = (x_1, y_1, z_1)$ and $v = (x_2, y_2, z_2)$ are two vectors in IR^3, define their inner product.

(5 Marks)

d) Let the set S of vectors in IR^3:

$S = \{\mu_1 = (1,2,1)\ \mu_2 = (2,1,-4)\ \mu_3 = (3,-2,1)\}$

Normalize S to obtain an orthonormal basis of IR^3 (5 Marks)