UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR DEGREE OF BACHELOR OF EDUCATION AND BACHELOR SCIENCE

COURSE CODE: **MAP 311**

COURSE TITLE: **REAL ANALYSIS II**

DATE: 08/12/2023 **TIME**: 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer question ONE and any other two questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following:

i) Continuous function (4 marks)

ii) Relatively compact set (2 marks)

iii) Open and closed sets (4 marks)

iv) Cauchy sequence (4 marks)

v) Accumulation points (4 marks)

b) Show that if $f \in C\ (X, Y)$ and X is compact then the image $f(X)$ is compact in Y (5 marks)

c) Suppose (X, d) is a metric space. Show that if { $E_1, E_2, \ldots, E_n$ } is any finite collection of closed subsets of X with respect to (X, d), then $\bigcup_{i=1}^{n} Ei$ is also closed in (X, d). (7 marks)

QUESTION TWO (20 MARKS)

a) Show that closed subsets of compact metric spaces are compact (10 marks)

b) Show that if $(X_i\ , d_i\ )$, i = 1,…, n are metric spaces. Then $X = X_1 \times X_2 \times \ldots X_n$ becomes a metric space with the metric d define by:

$d(x, y) := \sum_{i=1}^{n} d_i\ (\ x_i\ , y_i)$ for all $x = (x_i, \ldots . x_n)$ and $y = (y_i, \ldots . y_n)$ in X

(10 marks)

QUESTION THREE (20 MARKS)

a) Show that a sequence in a metric space (X, d) has at most one limit (10mks)

b) Show that if U is a subset of a metric space (X, d), then x ∈ U if and only if there exists a sequence (x_n) in U such that $x_n \to$ x as n → ∞ (10 marks).

QUESTION FOUR (20 MARKS)

a) Define the terms:

(i) Open and closed ball (4 mark)

(ii) Interior point (2marks)

(iii) Metric space (4 marks)

b)Show that for every non empty set A ⊆ X the map X → ℝ , x↦ dist(x, A), is continuous. (10 marks)

QUESTION FIVE (20 MARKS)

a) Let (X, d) be a metric space.

(i) Show that arbitrary union of open sets are open. (6 marks)

(ii) Show that intersection of open sets are open. (6 marks)

b) Show that if $f: X \longrightarrow Y$ and $g: Y \longrightarrow Z$ are continuous functions between metric spaces. Then the composition $g \ o \ f: X \longrightarrow Z$ is continuous (8 marks)