UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

COURSE CODE: **MAP 321**

COURSE TITLE: REAL ANALYSIS III

DATE: 27/4/2023 **TIME:** 9 AM - 11 AM

Answer question ONE and any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Write the Fourier series of the given function on the given interval

i. $f(x) = |x|, \quad -\pi \leq x \leq \pi$

ii. $f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ -1 & -1 \leq x \leq 0 \end{cases}$

b) Suppose that we have a series of sines and cosines which represents a given function on $[-L, L]$, say $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin(\frac{n\pi x}{L})$

What would such an equation tell us about $a_0, a_1, a_2, \dots \dots \dots, b_1, b_2, b_3 \dots\dots$?

c) Define the following terms

i. Right limit, left limit

ii. Sectionally continous

QUESTION TWO (20 MARKS)

a) State the theorem for convergence of Fourier series

b)

i. Find $f(x_{0+}), f(x_{0-}), f_{\mathbb{R}}'(x_0)$ and $f_L'(x_0)$ at each point x_0 in $[-L, L]$ where $f(x)$ has a discontinuity

ii. Also find $f(-L +), f(L_{-}), f_{\mathbb{R}}'(-L)$ and $f_L{}'(L)$

iii. In each of the following cases, determine the limit of the Fourier series of f(x) on $[-L, L]$

1) $f(x) = \begin{cases} 2 & -3 \leq x \leq -1 \\ |x| & -1 < x \leq 0 \\ x^2 & 0 < x < 2 \\ x + 5 & 2 < x \leq 3 \end{cases}$

2) $f(x) = \begin{cases} 1 - x & -4 \leq x < 0 \\ 2 & 0 < x < 3 \\ \sqrt{e^x} & 3 < x < 4 \end{cases}$

3) $f(x) = \begin{cases} 0 & -2 \leq x < 1 \\ 1 & 0 \leq x \leq 1 \\ 2 & 1 < x \leq 2 \end{cases}$

QUESTION THREE (20 MARKS)

a) Define the following terms

   i. Even function

   ii. Odd function

   iii. Fourier cosine series of a function f(x) on $[0, L]$

b)

   i. Define a Riemann integral

   ii. State the five properties of a Riemann integral

c) Given $f(x) = e^{2x}, 0 \leq x \leq 1$, find the sine expansion

QUESTION FOUR (20 MARKS)

a) Using the Riemann series for an integral derive the Fourier integral

b) Suppose that $\int_{-\infty}^{\infty} |f(t)| dt$ is finite, state the conditions under which its Fourier Integral converges

c) Given $f(x) = \begin{cases} e^{-x} \; for \; x \geq 0 \\ e^{x} \; for \; x \leq 0 \end{cases}$

   i. Show that $f(x)$ is finite

   ii. Find the Fourier integral for $f(x)$

   iii. Find the point of convergence for $f(x)$

QUESTION FIVE (20 MARKS)

a) State the

   i. Fourier cosine integral of $f(x)$

   ii. Fourier sine integral of $f(x)$

b) Given $f(x) = -e^{-x}$ for $x \geq 0$

   i. Find $\int_{0}^{\infty} |f(x)| dx$

   ii. Find the sine integral of $f(x)$