UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SUPPLEMENTARY/SPECIAL EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

COURSE CODE: **MAP 322**

COURSE TITLE: GROUP THEORY II

DATE: 15/08/23 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30MARKS)

a. Define the following
   i. Conjugacy class (2marks)
   ii. Centralizer (2marks)
   iii. P-groups (2marks)
   iv. Sylow p-subgroup (2marks)

b. Show that every nilpotent group is solvable (4 marks)

c. Show that if H is a proper subgroup of a nilpotent group G , then H is a proper subgroup of $N_G(H)$ (10marks)

d. Show that all finite abelian groups are soluble (8 marks)

QUESTION TWO (20MARKS)

a. Define the following sets
   i. Upper central series (2marks)
   ii. Nilpotent group (2 marks)
   iii. Central series (2marks)

b. Show that if G is the internal direct product of H and K, then G is isomorphic to the external direct product H×K (9marks)

c. Show that if G is a group, G acts on itself by conjugation: $g * x = gxg^{-1}$ for g, x∈ G (5marks)

QUESTION THREE (20MARKS)

a. State the following theorems
   i. Cauchy theorem (2marks)
   ii. Sylow's first theorem (2marks)
   iii. Sylow's second theorem (2 marks)

b. Show that any cyclic abelian group is isomorphic to z or z_n for some n. (7marks)

c. Show that every finite group G has a composition series (7marks)

QUESTION FOUR (20MARKS)

a. Define the following
   i. Maximal normal subgroup (2marks)
   ii. Composition series (3marks)

b. Show that every p-subgroup of G is contained in some sylow p- subgroup of G. (8marks)

c. Let p be prime, show that the center of a nontrivial finite p-group is nontrivial (7marks)

QUESTION FIVE (20MARKS)

a. Define the following
   i. External direct product (2marks)
   ii. Internal direct product (2marks)

b. State the Jordan – holder theorem (6marks)

c. Let p be prime. Show that the order of a finite p-group is pn for some n> 0 (5marks)

d. Show that a Sylow p-subgroup of G is unique if and only if it is normal in G. In particular it is unique if the group is abelian. (5 marks)