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**UNIVERSITY EXAMINATIONS  
2022/2023 ACADEMIC YEAR  
THIRD YEAR SECOND SEMESTER  
MAIN EXAMINATION**

**FOR THE DEGREE OF BACHELOR OF EDUCATION AND  
BACHELOR OF SCIENCE**

**COURSE CODE: MAP 323**

**COURSE TITLE: RING THEORY**

**DATE: 28/04/2023**

**TIME: 9 AM - 11 AM**

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

### QUESTION ONE (30 MARKS)

- a) Explain the meaning of the following terms as used in ring theory (2mks @)
- Euclidean Domain
  - Integral domain
  - Principal Ideal Domain
  - A Boolean ring
- b) Determine whether or not the following polynomials are irreducible over  $Z_5$
- $f(x) = x^3 + 2x^2 - 3x + 4$  (2 mks)
  - $g(x) = x^2 + 3x + 4$  (2 mks)
- c) Show that a Boolean ring  $B$ ,  $x^2 = x$  for each  $x \in B$  implies  $2x = 0$  (4 mks)
- d) Let  $x$  be a non zero element of a ring  $R$  with unity. Suppose there exists a unique  $y \in R$  such that  $xyz = x$ , show that  $xy = 1 = yx$ . (6 mks)
- e) State and prove the Euler's Theorem (8marks)

### QUESTION TWO (20 MARKS)

- a) Let  $R$  be a commutative ring with identity.
- Show that if  $e$  an idempotent element of  $R$ , then  $1 - e$  is also idempotent. (6 mks)
  - Show that if  $e$  is an idempotent element of  $R$  then  $R \cong Re \oplus R(1 - e)$  (14 mks)

### QUESTION THREE (20 MARKS)

- a) Let  $R$  be the ring of real numbers with unity, and let  $R[x]$  be the polynomial ring over  $R$ . Let  $J = (x^2 + 1)$  be the ideal in  $R[x]$  consisting of the multiples of  $x^2 + 1$ . Show that the quotient  $R[x]/J$  is the field of complex numbers. (12 mks)
- b) Let  $f: R \rightarrow S$  be a homomorphism of the ring  $R$  into a ring  $S$ . Show that the set  $\{f(a) | a \in R\}$  is a subring of  $S$ . (8 mks)

### QUESTION FOUR (20 MARKS)

- a) Find  $q(x)$  and  $r(x)$  in  $Z_5[x]$  if  $g(x) = 2x^3 + 3x^2 + 4x + 1$  is divided by  $f(x) = 3x + 1$ . (8 mks)

- b) Determine the idempotents, nilpotent elements and the units of the ring of integers modulo 10 ( $Z_{10}$ ) (6 mks)
- c) Find all cyclic subgroups of the group of units of the ring of integers modulo 24 ( $Z_{24}$ ) (6 mks)

**QUESTION FIVE (20 MARKS)**

- a) Show that the ring of Gaussian integers  $R = \{m + n\sqrt{-1} \mid m, n \in \mathbb{Z}\}$  is a Euclidean ring if we set  $\phi(m + n\sqrt{-1}) = m^2 + n^2$  (12 mks)
- b) Let  $A$  and  $B$  be ideals in  $R$  such that  $B \subseteq A$ . Show that  $R / A \cong (R / B) / (A / B)$  (8 mks)