UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY

COURSE CODE: MAT 121

COURSE TITLE: LINEAR ALGEBRA I

DATE: 12/4/2023 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Answer the following as either True or False, and justify your answer

If A and B are 2×2 matrices such that det(A)=-2 and det(B)=8, then
$\det(2A^T adj(B^{-1})) = 1$ (4marks)

b) Verify that the triangle with vertices $A(1, 1, 2)$, $B(1, 2, 3)$, and $C(3, 0, 3)$ is a right angled triangle. (5marks)

c) Find the angle between X=(0,1,1,0) and Y=(1,1,0,0) (4marks)

d) Let $A^{-1} = \begin{bmatrix} 0 & 4 & 4 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 5 & 3 \\ 0 & 1 & 2 & 6 \end{bmatrix}$, find det(A) (6marks)

e) Find all $x, y \in$ R such that the vectors $U = (x, 1, 3)$ and $V = (4, x + y, 9)$ are parallel. (4marks)

f) Show that if **x** and **y** are orthogonal unit vectors in $\mathbb{R}^n$, then $||4\boldsymbol{x} + 3\boldsymbol{y}|| = 5$. (4marks)

g) Let X and Y be vectors in $\mathbb{R}^n$, such that $\| X \| = \| Y \|$. Show that $X + Y$ and $X - Y$ are orthogonal vectors. (3marks)

QUESTION TWO (20 MARKS)

a) Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Find (14marks)

i. $Coef(A)$
ii. $det(adj(A))$
iii. $A^2 adj(A)$

b) Given the set $S = \{1,2,3\}$, state the inversions and the signs of its permutations (6marks)

QUESTION THREE (20 MARKS)

a) List 5 properties of matrix determinant (5marks)

b) Let $detA = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = -4$, and $B = \begin{bmatrix} 2a_3 & 2a_2 & 2a_1 \\ b_3 - a_3 & b_2 - a_2 & b_1 - a_1 \\ c_3 + 3b_3 & c_2 + 3b_2 & c_1 + 3b_1 \end{bmatrix}$, evaluate $|B|$ (6marks)

c) Evaluate the determinant of the matrix A via reduction to triangular form where
$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \\ -1 & 3 & 1 \end{bmatrix}$ (5marks)

d) Show that if **x** and **y** are in $\mathbb{R}^n$, then $\| x + y \| \leq \| x \| + \| y \|$ (4marks)

QUESTION FOUR (20 MARKS)

a) Let A be a non-singular 4×4 matrix with $|A^{-1}| = 3$. Find

i. $|\text{adj}(A)|$ (4marks)

ii. $|\frac{1}{2}A^T Adj(A^{-1})|$ (4marks)

b) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & 4 \\ 3 & 2 & 1 \end{bmatrix}$. Find the columns of AB as a linear combination of columns of A (6marks)

c) Solve the following linear system using Gauss Jordan method (6marks)

$x + 3y - z + w = 1$

$2x - y - 2z + 2w = 2$

$3x + y - z + w = 1$

QUESTION FIVE (20 MARKS)

a) Find the matrix A satisfying $\left(2A^T - 3\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\right)^T = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ (6marks)

b) Find a vector X, of length 4, in the opposite direction of $Y = (2, 2, -1)$. (5marks)

c) Let $A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 1 & 1 & -1 \end{bmatrix}$ (9marks)

i. Find B^{-1}

ii. Find C if $A = BC$