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UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 351**

COURSE TITLE: **ENGINEERING MATHEMATICS III**

DATE: 08/12/2023   **TIME:** 9:00 AM – 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer question ONE and any other two questions

TIME: 2 Hours

This Paper Consists of **3 Printed Pages**. Please Turn Over.

QUESTION ONE   (30 MARKS)

a) Evaluate $L\{(f(t)\} = \sin^2 t$ (4 marks)

b) Using the Laplace transforms find the solution to the initial value problem $y'' - y = 1$ $y(0) = 0$ $y'(0) = 1$ (5 marks)

c) Find the Fourier series of $f(x) = \begin{cases} 2 & -\Pi \le x \le 0 \\ 1 & 0 \le x \le \Pi \end{cases}$ (8 marks)

d) Evaluate $\lim\limits_{x \to 8} \dfrac{x^2 - 64}{x - 8}$ (3 marks)

e) Find the convolution of $f(t) = e^{-t}$ and $g(t) = \sin t$ (5 marks)

f) Find $L\{\frac{1}{t}(1 - e^t)\}$ (5 marks)

QUESTION TWO (20 MARKS)

a) Find $L\{t \sinh at\}$ (3 marks)

b) Evaluate Find $L^{-1}\left\{\dfrac{5}{s^3} + \dfrac{7}{s^4}\right\}$

(4 marks)

c) Write down the partial fraction of $F(s) = \dfrac{1}{(s-1)(s+2)(s+4)}$

hence determine $L^{-1}\left\{\dfrac{1}{(s-1)(s+2)(s+4)}\right\}$ (7 marks)

d) Find the unit normal to the surface $xy^3 z^2 = 4$ at $(-1, -1, 2)$
(6 marks)

QUESTION THREE   (20 MARKS)

a) Find the directional derivative of

$\Phi = x^2 yz + 4xz^2$ $at(1, -2, -1)$ $along$ $2i - j - 2k$ (6 marks)

b) Evaluate the value of $\lim\limits_{x \to 0} \dfrac{\sin x}{x}$ (4 marks)

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c) $L\{\cosh at\} = \dfrac{s}{s^2 - a^2}$ prove. (5 marks)

d) Given the function show that $f(x, y) = x^3 y^5$

$$\frac{\partial^2 f}{\partial x \, \partial y} = \frac{\partial^2 f}{\partial y \, \partial x}$$ (5 marks)

QUESTION FOUR (20 MARKS)

a) i) Given **P**= $x^2 + y^2 + z^2$ determine gradient of **P**
(3 marks)

ii) F=xy**i**+yz**j**+zx**k** find
div F (3marks)
curl F (4marks)

b) Given the function $f(x, y, z) = e^{x^2 + y^2 + z^2}$ find the total differential for this function.
(5 marks)

c) Work out
$$L^{-1}\left\{\frac{3s+5}{s^2+7}\right\}$$ (5 marks)

QUESTION FIVE 20 MARKS

a) Use Laplace transform to find the solution to the initial value problem
$y'' - y = 1 \quad y(0) = 0 \quad y'(0) = 1$ (6marks)

b) Evaluate $L^{-1}\left\{\dfrac{1}{2s-7}\right\}$ (3 marks)

c) Determine $\displaystyle\lim_{x \to -2} \frac{x^3 - 8}{x + 2}$
(4 marks)

d) Using Laplace transform, solve the initial value problem

$$y'' + 2y' + 2y = 5\sin t \quad y(0) = 0 \quad y'(0) = 0$$ (7 marks)

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