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**UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
FOURTH YEAR FIRST SEMESTER
MAIN EXAMINATION**

**FOR THE DEGREE OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE**

COURSE CODE: MAT 423/MAA 413

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATION II

DATE: 13/4/2023

TIME: 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Determine the stability of the system $\dot{x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} x$ (5 marks)

b) Show that there exist a unique solution to the differential equation

$$\frac{d^3 y}{dx^3} - 2 \frac{d^2 y}{dx^2} - \frac{dy}{dx} + 2y = 0, \text{ hence find the unique solution.} \quad (7 \text{ marks})$$

c) Linearize and hence solve the non-linear differential equation $y' = y^2 + 1$ at $y(0) = 1$ (6 marks)

d) Use matrix method to solve the following system of differential equations

$$X' = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (8 \text{ marks})$$

e) Use Picard's method to approximate the value of y when $x=0.1$ given that $y = 1$ when $x = 0$

$$\text{and } \frac{dy}{dx} = x + y. \quad (4 \text{ marks})$$

QUESTION TWO (20 MARKS)

a) Use elimination method to solve the system

$$2 \frac{dx}{dt} + \frac{dy}{dt} + x - y = 0 \quad (10 \text{ marks})$$

$$\frac{dx}{dt} + \frac{dy}{dt} + 9x = 9$$

b) Use row reduction method to solve the differential equation defined by

$(x^2 - 1)y'' - 2xy' + 2y = 0$ given that $y = x$ is a solution of the differential equation. (10 marks)

QUESTION THREE (20 MARKS)

Find the power series solution for the initial value problem

$$xy'' + y' + 2y = 0$$

$$y(1) = 2$$

$$y'(1) = 2$$

at the ordinary point $x = 1$

(20 Marks)

QUESTION FOUR (20 MARKS)

- a) State the condition for the following critical points to occur and in each case draw the phase portrait
- i) Node (2 marks)
 - ii) Saddle point (2 marks)
- b) Consider a nonlinear system

$$f(x) = \begin{bmatrix} x_1^2 - x_2^2 - 1 \\ 2x_2 \end{bmatrix} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix}$$

Analyze the system by

- i) Finding the critical points (4 marks)
- ii) Linearize the system and determine the type of critical point it has. Draw the phase portrait in each case. (12 marks)

QUESTION FIVE (20 MARKS)

- a) Find the general solution of the system $X' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} X$ (6 marks)
- b) Determine the respective fundamental matrix $x(t)$ given that $x(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (11 marks)
- c) Hence find $e^{\begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} t}$ (3 marks)