UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST-YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

COURSE CODE: **MAT 851**

COURSE TITLE: ODE I

DATE: 16/08/23 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

This Paper Consists of 3Printed Pages Please Turn Over

QUESTIONONE [20 MARKS]

(a) Define the following terms
   (i) Existence of a problem (2 mks)
   (ii) Uniqueness of a problem (2 mks)
   (iii) Orbit (2 mks)
   (iv) Invariant set (2 mks)

(b) By computing appropriate Lipschitz constant show that the following function satisfy Lipschitz conditions on the set S
$f(x,y) = 9x^2 + y^2$ on $S = \{(x,y)/\ |x| \leq 1,\ |y| \leq 1\}$ (5 mks)

(c) Given the equation $\begin{cases} x' = y \\ y' = -x \end{cases}$
   (i) Rewrite in polar coordinates (5 mks)
   (ii) Draw its phase portrait (2 mks)

QUESTION TWO [20 MARKS]

(a) Given the equation $(\boldsymbol{x},\boldsymbol{y})' = (\boldsymbol{x}^2 - \boldsymbol{x} - \boldsymbol{y}, \boldsymbol{x})$, show that the orign is asymptotically stable (6 mks)

(b) Given tha IVP

$$\begin{cases} \boldsymbol{y}'(\boldsymbol{x}) = \frac{e^{y^2-1}}{1-x^2y^2} \\ \boldsymbol{y}(-2) = \mathbf{1} \end{cases}$$

Find an interval on which a solution exists (14 mks)

QUESTION THREE [20 MARKS]

Consider the IVP $\boldsymbol{y}' = \mathbf{2}\boldsymbol{y} + \mathbf{1}, \boldsymbol{y}(\mathbf{0}) = \mathbf{2}$

(a) Show that all the successive approximationsΦ_0, Φ_1, Φ_3.... exists for all real x (5 mks)

(b) Compute the 1st 4 approximations (10 mks)

(c) Compute the exact solution (5 mks)

QUESTION FOUR [20 MARKS]

(a) Define a Lyapunov function (3 mks)

(b) Consider $\begin{cases} x' = -x + y \\ y' = -x - y^3 \end{cases}$

Show that $V(x,y) = x^2 + y^2$ is a strict Lyapunov function for $(0,0)$ (5 mks)

(c) Consider $\begin{cases} x' = 3x + y^3 \\ y' = -2y + x^2 \end{cases}$ and the function $V(x,y) = x^2 - y^2$ show that the orign is unstable (6 mks)

(d) Given the equation $\begin{cases} x' = y - xy^2 \\ y' = -x^3 \end{cases}$

Discuss the stability of the critical point at the orign (6 mks)

QUESTION FIVE [20 MARKS]

(a) Define (4 mks)

(i) A regular path

(ii) index

(b) State the Grobmann-Hartman theorem (4 mks)

(c) Consider the equation

$$\begin{cases} x^{l} = x^2 - y^2 \\ y^{l} = x^2 + y^2 \end{cases}$$

(i) Show that there is an invariant straight line containing (0,0). (5 mks)

(ii) Show that there are no periodic orbits, (5 mks)

(iii) Sketch the phase portrait. (2 mks)