UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 421/MAA 412

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATION I

DATE: 6/12/2023 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages Please Turn Over

QUESTION ONE (30 MARKS)

a. State the second order linear partial differential equation (P.D.E) in its general form. (2mks)

b. By citing an appropriate example in each case, distinguish between partial differential equation and ordinary differential equation. (4marks)

c. State the order and degree of the partial differential equations below.

i) $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=z+xy$

ii) $y\left\{\left(\frac{\partial z}{\partial x}\right)^2\right\}+\left(\frac{\partial z}{\partial y}\right)^2=z\left(\frac{\partial z}{\partial y}\right)$ (4 marks)

d. Classify the following partial differential equations as elliptic, parabolic of hyperbolic.

i) $(x+1)u_{xx}-2(x+2)u_{xy}+(3+x)u_{yy}=0$ (1mk)

ii) $f_{xx}+2f_{xy}+f_{yy}=0$ (1mk)

e. Solve $\frac{\partial^2 z}{\partial x^2}-a^2z=0$ under the conditions: $\frac{\partial z}{\partial x}=a\sin y$ and $z=0$, when $x=0$. (6mks)

f. Apply Lagrange method to solve $\left(\frac{y^2z}{x}\right)p+xzq=y^2$ (6mks)

g. Form a partial differential equation by eliminating arbitrary constant in the equation

$z=ax+by+a^2+b^2$ (3mks)

h. Form a partial differential equation whose solution is $\varphi\left(x+y+z,x^2+y^2-z^2\right)=0$

(3mks)

QUESTION TWO (20 MARKS)

a. Solve $D^3-3D^2D^1+4\left(D^1\right)^3=e^{x+2y}$ (6mks)

b. Form a partial differential equation by eliminating the arbitrary function in the equation

$y=f(x-at)+F(x+at)$ (4mks)

c. By choosing appropriate multipliers, solve

$\frac{y-z}{yz}p+\frac{z-x}{zx}q=\frac{x-y}{xy}$ (6mks)

d. Solve the non-linear partial differential equation $p^2+q^2=npq$ (4mks)

QUESTION THREE (20 MARKS)

a. Apply Charpit's method to solve non-linear partial differential equation $px + qy = pq$ (10mks)

b. Show that the equations: $xp - yq = 0$ and $xup + yuq = 2xy$ are compatible. Hence, solve them. (10mks)

QUESTION FOUR (20 MARKS)

a) Find the complete integral of $z^2 + zu_z - u_x^2 + u^2_y = 0$ by Jacobi method. (5mks)

b) Solve the equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + 2z = 0$ given $z = e^x$ and $\frac{\partial z}{\partial x} = 0$ when $x = 0$ (10marks)

c) Solve by direct integration method the equation $\frac{\partial^2 z}{\partial x^2} = x + y$ given that $z = y^2$ when $x = 0$ and $\frac{\partial z}{\partial x} = 0$ (5mks)

QUESTION FIVE (20 MARKS)

a) Transform the following partial differential equation to canonical form and solve using method of characteristics. $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$ (10marks)

b) Solve $z = px + qy - 2\sqrt{pq}$. (5mks)

c) Solve the Pfaffian differential equation $ydx + xdy + 2zdz = 0$ (5mks)