MAIN UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER FOR THE DEGREE OF

BACHELOR OF SCIENCE IN PHYSICS

COURSE CODE: SPC 313

COURSE TITLE: MATHEMATICAL PHYSICS I

DATE: 6/12/2023 **TIME: 9:00-11:00AM**

INSTRUCTIONS TO CANDIDATES

Answer question ONE and any TWO of the remaining.

Time: 2 hours

QUESTION ONE (30 MARKS)

a) Find the characteristic equation and the eigen values of the matrix (3 marks)

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

b) Find the product **AB**, given that $A = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}$ (3 marks)

c) Given two vectors, $\vec{A} = 2i + -3j + 7k$ and $\vec{B} = 5i + j + 2k$, find:

(i) $|\vec{A}|$ (2 marks)

(ii) $|\vec{B}|$ (2 marks)

(iii) A unit vector $\hat{A}$ pointing in the direction of $\vec{A}$ (2 marks)

(iv) A unit vector $\hat{B}$ pointing in the direction of $\vec{B}$ (2 marks)

d) Find the angle between vector $a = \begin{pmatrix} 3 & 1 & 5 \end{pmatrix}$ and $b = \begin{pmatrix} 2 & 1 & 3 \end{pmatrix}$ (3 marks)

e) Find the area of a parallelogram defined by coordinates $(0, \ 0, \ 0)$, $(3, \ 1, \ 4)$ and $(2 \ 1, \ 3)$ (3 marks)

f) Find the gradient of U in $U = r^2$ (2 marks)

g) Assuming that **A**, **B** and **C** are 2 x 2 matrices, show that **AB** = **AC** and A is invertible, then **B** = **C** (2 marks)

h) Find the orthogonal matrix (in standard basis) that implements reflection on a plane with equation: $2x_1 + 3x_2 + x_3 = 0$ (3 marks)

i) Given that A= $2i + 3j + k$, B = $-i + j$ and C = $-2i + 2j$, find **A**. (**B** x **C**) (3 marks)

QUESTION TWO (20 MARKS)

a) Figure 1 shows two force vectors $\vec{A}$ and $\vec{B}$, such that $|\vec{B}| = 2|\vec{A}|$ have a resultant $\vec{C} = \vec{A} + \vec{B}$ of magnitude 26.5 N, which makes an angle of 41° with respect to the smaller vector $\vec{A}$.

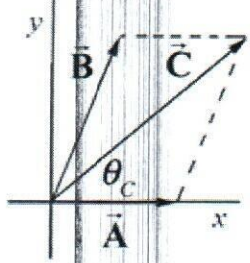

Figure 1

Find the magnitude of each vector and angle between them (5 marks)

b) Show that the set {1, cos *x*, cos 2*x*, ...} is orthogonal on the interval [-Π, Π]. (5 marks)

c) Find the norm of each function in the orthogonal set given in (b) (10 marks)

QUESTION THREE (20 MARKS)

a) Find *c* and *d* so that the following matrix is unitary: (7 marks)

$$\begin{bmatrix} \frac{1}{\sqrt{7}}(1+2i) & c \\ \frac{1}{\sqrt{7}}(1-i) & d \end{bmatrix}$$

b) A 2 x 2 real symmetric matrix A has eigenvalues 1 and 3.(2, -3) is an eigenvector corresponding to the eigenvalue 1.

(i) Find an eigenvector corresponding to the eigenvalue 3. (6 marks)

(ii) Find A (7 marks)

QUESTION FOUR (20 MARKS)

a) In the eigenvector equation A**X** = λ**X**, the operator **A** is given by $A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$,

(i) Find the eigenvalues of λ (3 marks)

(ii) Find the eigenvector **X**, hence the modal matrix C and its inverse C^{-1} (10marks)

(iii) The product $C^{-1}AC$ (2 marks)

b) Prove that the characteristic roots of a Hermitian matrix are real (5 marks)

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

b) Find the product **AB**, given that $A = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}$ (3 marks)

c) Given two vectors, $\vec{A} = 2i + -3j + 7k$ and $\vec{B} = 5i + j + 2k$, find:

(i) $|\vec{A}|$ (2 marks)

(ii) $|\vec{B}|$ (2 marks)

(iii) A unit vector $\hat{A}$ pointing in the direction of $\vec{A}$ (2 marks)

(iv) A unit vector $\hat{B}$ pointing in the direction of $\vec{B}$ (2 marks)

d) Find the angle between vector $a = (3 \quad 1 \quad 5)$ and $b = (2 \quad 1 \quad 3)$ (3 marks)

e) Find the area of a parallelogram defined by coordinates $(0, \; 0, \; 0)$, $(3, \; 1, \; 4)$ and $(2 \;\; 1, \; 3)$ (3 marks)

f) Find the gradient of U in $U = r^2$ (2 marks)

g) Assuming that **A, B** and **C** are 2 x 2 matrices, show that **AB** = **AC** and A is invertible, then **B** = **C** (2 marks)

h) Find the orthogonal matrix (in standard basis) that implements reflection on a plane with equation: $2x_1 + 3x_2 + x_3 = 0$ (3 marks)

i) Given that A= $2i + 3j + k$, B = $-i + j$ and C = $-2i + 2j$, find **A**. (**B** x **C**) (3 marks)