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**UNIVERSITY EXAMINATIONS  
2022/2023 ACADEMIC YEAR**

**THIRD YEAR SECOND SEMESTER  
SPECIAL/SUPPLEMENTARY EXAMINATIONS**

**FOR THE DEGREE OF BACHELOR OF SCIENCE (PHYSICS)**

**COURSE CODE:** SPC 321

**COURSE TITLE:** QUANTUM MECHANICS I

**DATE:** 10/8/2023

**TIME:** 8:00-10:00AM

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**INSTRUCTIONS TO CANDIDATES**

**TIME:** 2 HOURS

**Answer question ONE and any TWO of the remaining**

**QUESTION ONE [30 MARKS]**

a) Prove that if the commutator  $[\hat{A}, \hat{B}] = 1$  then:-  $[\hat{A}, \hat{B}^2] = 2\hat{B}$ . [3 marks]

b) If the operator  $\frac{d^2}{dx^2}$  operates on the wave function  $\psi = A \cos nx$  find the corresponding eigen value. [3 marks]

c) The state of a free particle is described by the following wave function:- [4 marks]

$$\psi(x) = \begin{cases} 0 & \text{for } x < -3a \\ C & \text{for } -3a < x < a \\ 0 & \text{for } x > a \end{cases}$$

Find C and the probability of finding the particle in interval  $[0, a]$ .

d) A particle moving in x-axis has a wave function given by:- [3 marks]

$$\psi(x) = \begin{cases} ax & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases} \text{ find } \langle x \rangle$$

e) The one- dimensional time-independent Schrödinger equation is given by [4 marks]

$$\left(-\frac{\hbar^2}{2m}\right)\left(\frac{d^2\psi(x)}{dx^2}\right) + V(x)\psi(x) = E\psi(x)$$

give the meaning of the symbols in this equation.

f) Given that:-  $J_x = \left(\frac{\hbar}{2mi}\right)\left[\psi^* \frac{d\psi}{dx} - \left(\frac{d\psi}{dx}\right)\psi^*\right]$ , show that  $J_x = \left(\frac{\hbar k}{m}\right)(A^2 - B^2)$  [6 marks]

$$\text{if } \psi = Ae^{ikx} + Be^{-ikx}$$

g) Consider a one-dimensional particle which is confined in a region  $0 < x < a$  [4 marks]

whose wave function is  $\psi(x, t) = \sin\left(\frac{\pi x}{a}\right) \exp(-i\omega t)$ . Find the potential energy  $V(x)$  of the region.

h) Show that  $i[\hat{A}, \hat{B}]$  will be Hermitian if  $\hat{A}$  and  $\hat{B}$  are Hermitian operators. [3 marks]

**QUESTION TWO [20 MARKS]**

a) What boundary conditions do wave functions obey? [2 marks]

(b) A particle confined to a one dimensional potential well has a wave function given by:-

$$\psi(x) = \begin{cases} 0 & \text{for } x < -L/2 \\ A \cos\left(\frac{3\pi x}{L}\right) & \text{for } -L/2 \leq x \leq L/2 \\ 0 & \text{for } x > L/2 \end{cases}$$

- (i) Sketch the wave function  $\psi(x) \approx \cos\left(\frac{3\pi x}{L}\right)$  [3 marks]
- (ii) Calculate the normalization constant A [5 marks]
- (iii) Calculate the probability of finding the particle in the interval  $-L/4 \leq x \leq L/4$  [5 marks]
- (iv) Using the Schrödinger equation  $\left(-\frac{\hbar^2}{2m}\right)\left(\frac{d^2\psi}{dx^2}\right) = E\psi$  show that the energy E corresponding to this wave function is given by:-  $\frac{9\pi^2\hbar^2}{2mL^2}$ . [5 marks]

**QUESTION THREE [20 MARKS]**

A particle moving in one dimension is in a stationary state whose wave function is given by:-

$$\psi(x) = \begin{cases} 0 & \text{for } x > -a \\ A\left(1 + \cos\frac{\pi x}{a}\right) & \text{for } -a \leq x \leq a \\ 0 & \text{for } x > a \end{cases}$$

- a) Find the magnitude of A so that  $\psi(x)$  is normalized. [5 marks]
- b) Evaluate  $\Delta x$  and  $\Delta p$  hence verify that  $\Delta x \Delta p \geq \frac{\hbar}{2}$ . [15 marks]

**QUESTION FOUR [20 MARKS]**

A particle of mass m, moves freely inside an infinite potential well of length a, has an initial wave function given by:-

$$\psi(x, 0) = \frac{A}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}} \sin\left(\frac{3\pi x}{a}\right) + \frac{1}{\sqrt{5a}} \sin\left(\frac{5\pi x}{a}\right)$$
 where A and a are real

constants,  $a=2$  and  $\phi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$ .

- a) Find A so that  $\psi(x, 0)$  is normalized [4 marks]
- b) What are the values of measurements of energy and their corresponding probabilities? [7 marks]
- c) Calculate the average energy. [3 marks]
- d) Find the wave function  $\psi(x, t)$  at any later time t. [3 marks]
- e) Determine the probability of finding the system at any later time t in state  $\varphi(x, t) = \sqrt{\frac{2}{a}} \sin\left(\frac{5\pi x}{a}\right) e^{-iE_5 t/\hbar}$ . [3 marks]

**QUESTION FIVE [20 MARKS]**

- a) The wave function of a hydrogen atom is given by:-  $\psi = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$ , find  $\langle r \rangle$  and  $\langle r^2 \rangle$  for an electron in the ground state express your answer in terms of Bohr's radius (a) [6 marks]



- b) Find  $\langle x \rangle$  and  $\langle x^2 \rangle$  for an electron in the ground state of a hydrogen atom if  $r^2 = x^2 + y^2 + z^2$ . [2 marks]
- c) Find  $\langle x^2 \rangle$  in the state  $n = 2, l = 1$  and  $m = 1$  if  $x = r \sin \theta \cos \phi$  [6 marks]
- d) Find the probability that an electron in the ground state of a hydrogen atom will be found inside the nucleus? [6 marks]