UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER
SUPPLEMENTARY/SPECIAL EXAMINATIONS

FOR THE DEGREE IN BSC (PHYSICS)

COURSE CODE: SPC 411

COURSE TITLE: QUANTUM MECHANICS II

DATE: 16/11/2022 **TIME**: 2:00PM-4:00PM

INSTRUCTIONS TO CANDIDATES

TIME: 2 HOURS

Answer question ONE and any TWO of the remaining

QUESTION ONE [30 MARKS]

a) Show that $(\sigma.\boldsymbol{A})(\sigma.\boldsymbol{B}) = \boldsymbol{A}.\boldsymbol{B} + i\sigma(\boldsymbol{A}x\boldsymbol{B})$ where **A** and **B** are vectors and $\sigma's$ are Pauli's matrices [5 marks]

b) Find the Eigen function of the operator:-$\hat{L}_x =- i\hbar\frac{d}{d\varphi}$. [5 marks]

c) A particle of mass m is trapped in a potential well of:-$V = \frac{1}{2}m\omega^2x^2$. Use variational method with the normalized trial wave function of:- $(1/\sqrt{a})cos(\pi x/2a)$ in the limits $-a < x < a$, find the best value of a. [5 marks]

d) The first Born approximation for elastic scattering amplitude is:-$f =- \left(\frac{2\mu}{q\hbar^2}\right)\int V(r)e^{iq.r}d^3r$ show that $V(r)$ is spherically symmetric and it reduces to:- $f =- \left(\frac{2\mu}{q\hbar^2}\right)\int rsin(qr)V(r)dr.$ [5 marks]

e) Show that:- $[\hat{L}^2, \hat{L}_x]\psi = 0$ [5 marks]

f) Calculate the scattering angle in laboratory frame of reference between two photons if it is 15^0 in the centre of mass frame. [5 marks]

QUESTION TWO [20 MARKS]

a) For Pauli's spin matrices σ_x, σ_y and σ_z obtain the commutators:- $[\sigma_x, \sigma_y]$, $[\sigma_x, \sigma_z]$ and $[\sigma_y, \sigma_z]$ [15 marks]

e) Use WKB approximation to find the allowed energies of the general power law potential $V(x) = \alpha|x|^\nu$ where $\nu = 2$ and it's a positive number. [5 marks]

QUESTION THREE [20 MARKS]

a) An electron is in spinor state $\chi = A\begin{bmatrix}3i\\4\end{bmatrix}$ hence:-

i) Determine the normalization constant A. [2 marks]

ii) Find the expectation values $\langle S_x\rangle$, $\langle S_y\rangle$ and $\langle S_z\rangle$. [6 marks]

iii) Find the uncertainties $\sigma^2_{S_x}$, $\sigma^2_{S_y}$and $\sigma^2_{S_z}$. [6 marks]

b) For a harmonic oscillator $V(x) = \frac{1}{2}kx^2$, the allowed energies are $E_n = (n + 1/2)\hbar\omega, (n = 0,1,2, ...)$, where $\omega = \sqrt{k/m}$. Now suppose the spring constant increases slightly by $k \to (1 + \epsilon)k$. Find the exact energies, trivial in this case. Expand your power series in ϵ, up to second order. [6 marks]

QUESTION FOUR [20 MARKS]

Show that in spherical coordinates the angular momentum operator is given by: - $\hat{L}_x =- i\hbar[- sin\phi\frac{\partial}{\partial\theta} - cot\theta cot\phi\frac{\partial}{\partial\phi}]$.

QUESTION FIVE [20 MARKS]

a) Find the energy levels and wave functions of a system of four distinguishable spinless particles placed in a an infinite potential well of [8 marks] size a. use this result to infer the energy and the wave function of the ground state and the first excited state

b) Find the ground state energy and wave function of a system of N-interacting identical particles that are confined to a one dimensional infinite well when the particles are bosons and spin $\frac{1}{2}$ fermions. [12 marks]