UNIVERSITY EXAMINATIONS
2023/2024 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE IN BSC (PHYSICS)

COURSE CODE: SPC 411

COURSE TITLE: QUANTUM MECHANICS II

DATE: 19/12/2023 **TIME:** 9:00-11:00AM

INSTRUCTIONS TO CANDIDATES

TIME: 2 HOURS

Answer question ONE and any TWO of the remaining

QUESTION ONE [30 MARKS]

a) For Pauli's spin matrices, show that:- $\sigma_x\sigma_y\sigma_z = i$. [5 marks]

b) Find the Eigen function of the operator:-$\hat{L}_x = -i\hbar\frac{d}{dx}$. [5 marks]

c) Show that the spin-orbit interaction energy is given by [5 marks]

$H' = -\frac{e^2}{2M^2c^2r^3}\frac{1}{2}(J^2 - L^2 - S^2)$.

d) For a state of angular momentum $|l,m\rangle$ show that $\Delta\hat{L}_x\Delta\hat{L}_y = \frac{m\hbar^2}{2}$. [5 marks]

e) Consider elastic scattering of $50MeV$ neutrons from a nucleus. The phase shifts measured in an experiment are $\delta_0 = 96^0$, $\delta_1 = 72^0$, $\delta_2 = 60^0$, $\delta_3 = 35^0$, $\delta_4 = 18^0$ and $\delta_5 = 5^0$ where $\delta_l = 0$ for $l \geq 6$. Find the total cross section and estimate the radius of the nucleus. [5 marks]

f) Calculate the scattering angle in laboratory frame of reference between two photons if it is 10^0 in the centre of mass frame. [5 marks]

QUESTION TWO [20 MARKS]

a) Use variational method to estimate the ground state energy of a hydrogen atom. Use trial wave function $\psi(r,\theta,\phi) = e^{-r/\alpha}$ and $H = \frac{\hbar^2}{2m}\nabla^2 + \frac{e^2}{r}$ [10 marks]

b) Use the WKB approximation method to estimate the energy levels of the states of an electron that is bound to the Ze nucleus with Coulomb field of $V(r) = -Ze^2/r$.

QUESTION THREE [20 MARKS]

a) An electron is in spinor state $\chi = A\begin{bmatrix}3i\\4\end{bmatrix}$ hence:-

i) Determine the normalization constant A. [2 marks]

ii) Find the expectation values $\langle S_x\rangle$, $\langle S_y\rangle$ and $\langle S_z\rangle$. [5 marks]

iii) Find the uncertainties $\sigma_{S_x}^2$, $\sigma_{S_y}^2$ and $\sigma_{S_z}^2$. [5 marks]

b) The 2s wave function of a hydrogen atom is given by [8 marks]

$\psi_{200}(r,\theta,\phi) = \frac{1}{\sqrt{4\pi}}\left(\frac{1}{2a_0}\right)^{3/2}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$ with perturbation $V(r) = \frac{\lambda}{r^2}$

use the perturbation theory to calculate the first order shift in energy.

QUESTION FOUR [20 MARKS]

a) If $\hat{L}_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$, $x = rsin\theta cos\varphi$, $y = rsin\theta sin\varphi$, $\frac{\partial}{\partial y} = sin\theta sin\varphi\frac{\partial}{\partial r} + \frac{cos\theta sin\varphi}{r}\frac{\partial}{\partial\theta} - \frac{cos\varphi}{rsin\theta}\frac{\partial}{\partial\varphi}$ and $\frac{\partial}{\partial x} = sin\theta cos\varphi\frac{\partial}{\partial r} + \frac{cos\theta cos\varphi}{r}\frac{\partial}{\partial\theta} - \frac{sin\varphi}{rsin\theta}\frac{\partial}{\partial\varphi}$. Show that $\hat{L}_z = i\hbar\frac{\partial}{\partial\varphi}$ [12 marks]

b) Show that: - $\vec{L}X\vec{L} = i\hbar\vec{L}$. [8 marks]

QUESTION FIVE [20 MARKS]

a) Calculate the differential cross section in the first Born approximation for a Coulomb potential $V(r) = Z_1Z_2e^2$ where Z_1e and Z_2e are the [10 marks]

charges of the projectile and target respectively. Consider scattering of an alpha particle ($Z_1 = 2$ and $A_1 = 4$) from a gold nucleus particle ($Z_2 = 79$ and $A_2 = 197$), if the scattering angle of the alpha particle in lab frame is $\theta_1 = 60^0$ find the scattering angle in CM frame.

b) A system has three non-interacting particles of half spin and confined in a one dimensional potential well of width 1 whose potential is given by $V(x) = \begin{cases} 0 & 0 \leq x \leq a \\ \infty & x < 0, x > a \end{cases}$. Generally the energy and wave function of the system are given by $E_n = \frac{n\hbar^2\pi^2}{2ma^2}$ and $\psi_n(x_i) = \sqrt{\frac{2}{a}} sin\left(\frac{n\pi x_i}{2}\right)$ respectively. Determine the ground state, the first excited state and the second excited states energies and their corresponding wave functions of the system [10 marks]