UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST-YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION

AND BACHELOR OF SCIENCE

COURSE CODE: STA 112/STA 142

COURSE TITLE: INTRODUCTION TO PROBABILITY

DATE: 15/08/2023   **TIME**: 11:00 AM – 1:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **3 Printed Pages**. Please Turn Over.

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS.

QUESTION ONE (30 MARKS)

1. (a) Explain the following terms as used in set theory (3 marks)
    i. Set
    ii. Element
    iii. Power set

    (b) Write down the power sets of A if $A = \{1, 2, 3\}$ (3 marks)

    (c) Diferentiate between a discrete random variable and a continuous random variable and give an example in each case. (3 marks)

    (d) A random variable X has the probability distribution below

X	0	1	2	3	4	5	6	7	8
P(X=x)	a	3a	5a	7a	9a	11a	13a	15a	17a

    i. Determine the value of a (2 marks)
    ii. Find (4 marks)
        A. $P(X < 3)$
        B. $P(X \geq 3)$
        C. $P(0 < X < 5)$

    (e) A committee of 4 people need to be selected from 5 women and 7 men. How many ways can the committee be selected if atleast 3 women must be included. (4 marks)

    (f) If A and B are any two events in S. Show that
    $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (5 marks)

    (g) X be random variable with pdf

    $$f(x) = \begin{cases} \frac{x}{10}, & x=1,2,3,4 \\ 0, & elsewhere \end{cases}$$

    Compute $E(X)$, $Var(X)$ and $E(5X^3 - 2X^2)$ (6 marks)

QUESTION TWO (20 marks)

(a) The table below shows probability distributions of means obtained by some students in an examination

X	9	14	17	20	22	26
P(X=x)	0.0811	a	0.1872	a	0.2162	0.1622

    i. Find the value of a

    ii. Find the expected mark and variance of the score

    iii. What is the probability that a random picked student from this class scored

        A. more than 20 marks

        B. between 14 and 21 marks inclusive

(b) Prove the Bayes theorem

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{\sum_{i=1}^{n} P(A_i)P(B|A_i)}$$

QUESTION THREE (20 marks)

(a) Three newspapers A, B and C are published in a town. It is estimated from survey that 20 percentage read A, 16 percentage read B and 14 percentage read C, 8 percentage read A and B, 5 percentage read A and C and 2 percentage read all the three papers. What is the probability that a randomly chosen person:

    i. does not read any paper

    ii. reads A but not B

    iii. does not read C

    iv. reads only one of these papers

    v. reads only two of these papers

(b) Let X have the pdf

$$f(x) = \begin{cases} \frac{1}{2}(x+1), & -1 < x < 1 \\ 0, & elsewhere \end{cases}$$

Find

    i. $Var(X)$

    ii. $Var(5X + 10)$

QUESTION FOUR (20 marks)

(a) Consider tossing two fair dice. Let X denote the outcome of the two dice and Y the absolute difference. Calculate the expected value of X and Y.

(b) Let X be a random variable with a distribution function

$$f(x) = \begin{cases} Ax & 0 \leq x \leq 5 \\ A(10-x), & 5 < x \leq 10 \\ 0. & elsewhere \end{cases}$$

    i. Find A such that f(x) is a pdf

ii. Find $P(2.5 \leq X \leq 7.5)$