UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

PECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

COURSE CODE: STA 211

COURSE TITLE: PROBABILITY AND STATISTICS

DATE: 14/8/2023 **TIME:** 11:00 A.M – 1:00 P.M

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

1. (a) A continuous random variable X has the probability density function $f(x)$ given by $f(x) = kx^2(1-x)$ for $0 \leq x \leq 1$, $f(x) = 0$ elsewhere, where k is a constant. Determine

   i. the value of k (1 mks)

   ii. $E(X)$ and $var(X)$ (4 mks)

   iii. $P(X < E(X))$ (2 mks)

   (b) Show that $var(CX) = C^2 var$ where C is a constant (2 mks)

   (c) i. Define moment generating function (1 mk)

   ii. Show that if X and Y are independent random variables with moment generating functions $M_X(t)$ and $M_Y(t)$ and $Z = X + Y$ then $M_Z(t) = M_X(t)M_Y(t)$ (3 mks)

   (d) State and explain any four assumptions of the binomial distribution (4 mks)

   (e) The moment generating function of a random variable X is $e^{(e^t-1)}$. Determine $P(\mu - 2\sigma < x < \mu + 2\sigma)$ (4 mks)

   (f) An urn contains 6 red and 4 white marbles. Three marbles are drawn at random without replacement. Find that

   i. at least two white marbles were drawn (2 mks)

   ii. exactly two white marbles were drawn (2 mks)

   (g) A non-normal distribution representing the number of trips performed by trucks per week in an oil field has a mean of 100 trips and a variance of 121 trips. A random sample of 36 trucks is taken from the non-normal population. Using the central limit theorem, calculate the probability that the sample mean is

   i. greater than or equal to 105 trips (2 mks)

   ii. between 101 and 103 trips (2 mks)

QUESTION TWO (20 marks)

2. (a) Let

$$P(X=x)=\begin{cases}(\frac{1}{2})^x, & x=1,2,3,...\\ 0, & elsewhere\end{cases}$$

be the pdf of X. Find the moment generating function, mean and variance of X. (10 mks)

(b) At Nakumatt Supermarket, 60 percent of the customers pay by the credit card. Find the probability that in a randomly selected sample of 10 customers:

i. exactly two customers pay by credit card (2 mks)

ii. more than seven customers pay by credit card (2 mks)

(c) Show that the expected value and variance of a random variable whose moment generating function is $M_X(x)=\frac{\lambda}{\lambda-t}$ is λ and $\frac{1}{\lambda^2}$ respectively. (6 mks)

QUESTION THREE (20 marks)

3. (a) Suppose X is a geometric random variable with the parameter P. Show that:

i. $E(X)=\frac{q}{p}$ (4 mks)

ii. $Var(X)=\frac{q}{p^2}$ (4 mks)

iii. $M_X(t)=\frac{p}{(1-qe)}$ (4 mks)

(b) If Y is a random variable having a students t distribution with k degrees of freedom, show that $E(X)=0$ for $k>1$ and $Var(X)=\frac{k}{k-2}$ for $k>2$ (8 mks)

QUESTION FOUR (20 marks)

4. (a) Given

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & elsewhere \end{cases}$$

Find the mean and variance of X (8 mks)

(b) A random variable X is normal with $\mu = 50$ and $\sigma = 10$. Compute the $P(45 < x < 62)$ (4 mks)

(c) Suppose X is a negative binomial random variable with parameter P. Compute its moment generating function and hence find the mean of X. (8 mks)

QUESTION FIVE (20 MARKS)

5. (a) A company manufactures resistors with a mean resistance of 81 ohms and a standard deviation of 8 ohms. Using the central limit theorem, find the probability that a random sample of size 16 resistors will have an average resistance less than 75 ohms. (5 mks)

(b) The MGF of a random variable X given by $M_X(t) = e^{(5t+2t^2)}$. Calculate $p(9 < X < 11)$ (7 mks)

(c) Suppose x is a negative binomial random variable with parameter P. Compute its MGF and hence find the mean of X (8 mks)