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UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: STA 311/STA 341

COURSE TITLE: THEORY OF ESTIMATION

DATE: 24/08/2023 **TIME:** 11:00 AM - 1:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS.

QUESTION ONE (30 MARKS)

1. (a) Differentiate between Point and Interval estimation. (3 mks)

   (b) A random sample $x_1, ..., x_5$ of size $n = 5$ is drawn from a normal population with unknown mean μ and known σ^2. Consider the following estimators of the population mean μ.

$$t_1 = \bar{x}$$
$$t_2 = \frac{x_1 + x_2}{2} + x_3$$
$$t_3 = \frac{2x_1 + x_2 + \lambda x_3}{3},$$

   where λ is such that t_3 is unbiased for μ

   i. Are t_1, t_2 unbiased?

   ii. State with reasons the best estimator. (6 mks)

   (c) Let $(X_1, \ldots, X_n)$ be a random sample of a Poisson random variable X with unknown parameter λ. Show that $\Lambda_1 = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $\Lambda_2 = \frac{1}{2}(X_1 + X_2)$ are both unbiased estimators of λ, which estimator is more efficient? (5 mks)

   (d) If $x_1, ..., x_n$ are the values of a random sample from an exponential population, find the maximum likelihood estimator of its parameter θ (4 mks)

   (e) i. When is an estimator said to be sufficient (2 mks)

   ii. Let $x_1, \ldots, x_n$ be a random sample from a population with pdf

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1}, x > 0 \\ 0, elsewhere \end{cases}$$

   Show that $t_i = \Pi_{i=1}^{n} X_i$ is a sufficient statistic for θ. (4 mks)

   (f) In a simple random sample of 600 men from city A, 450 are found to smokers. In another random sample of 900 men from city B, 450 are found to be smokers. Construct a 99 percent confidence interval for the difference in the population proportions. (6 mks)

QUESTION TWO(20 MARKS)

2. (a) Let $x_1, ..., x_n$ be a random sample from the gamma distribution with pdf:

$$f(x; m, \lambda) = \begin{cases} \frac{\lambda(\lambda x)^{m-1}}{\Gamma(m)} e^{-\lambda x}, x > 0, \\ 0, \quad otherwise \end{cases}$$

Assume m is known, obtain the ML estimator for λ (10 mks)

(b) Show that if $\lim_{n\to\infty} E(\Theta_n) = \theta$ and $\lim_{n\to\infty} var(\Theta_n) = 0$ then the estimator Θ_n is consistent. (5 mks)

(c) Let $x_1, x_2, ..., x_n$ be a random sample from a population given by

$$f(x) = \begin{cases} 1, \quad \beta - \frac{1}{2} < x < \beta + \frac{1}{2} \\ 0, \quad elsewhere \end{cases}$$

prove that $\bar{X}$ is a consistent estimator for β. (5 mks)

QUESTION THREE (20 MARKS)

3. (a) State precisely as possible the properties of maximum likelihood estimators (2 mks)

(b) A random sample of 50 students out of the total 200 showed a mean of 75 and a standard deviation of 10.

i. What are the 95 percent confidence limits for estimates of the mean of 200 students? (5 mks)

ii. With what degree of confidence could we say that the means of all the 200 students is 75 ± 1? (5 mks)

(c) Suppose $x_1, ..., x_n$ is a random sample from a population with $X \sim N(\mu, \sigma^2)$, σ^2 known. Use Cramer-Rao inequality to find UMVUE of μ (8 mks)

QUESTION FOUR (20 MARKS)

4. (a) i. When is an estimator t_n of θ said to be sufficient? (2 mks)

ii. For a normal population $N(\mu, \sigma^2)$ if σ^2 is known, prove that $\bar{X}$ is sufficient estimator for μ (5 mks)

iii. If μ is known, prove that s^2 is not a sufficient estimator for σ^2 and hence find its sufficient estimator. (3 mks)

(b) X is a binomial random variable with parameters n (known) and p (unknown). Given a random sample of N observations of X (10 mks)

i. Compute the method of moments estimator for p

ii. What will be the method of moments estimator for n and p when both are unknown

QUESTION FIVE(20 MARKS)

5. (a) A random sample is taken from a normal population with mean 0 and variance σ^2. Examine if $s^2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2$ is MVUE for σ^2 (8 mks)

(b) A random sample is picked from a population whose distribution is given by:

$$f(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$$

for $x = 0, 1, 2, \ldots, \infty$

Taking $\Psi(\lambda) = e^{-\lambda}$, find Cramer-Rao lower bound for t, where t is unbiased estimator for $\Psi(\lambda)$ (7 mks)

(c) Let $x_1, \ldots, x_n$ be iid random variable from a Bernoulli distribution with parameter P. Show that $T = \sum_i^n X_i$ is a sufficient statistic (5 mks)