UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: STA 426

COURSE TITLE: LARGE SAMPLE

DATE: 10/08/2023 **TIME**: 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

1. (a) State and explain probability tools for establishing consistency and asymptotic normality of estimators. (4 mks)

   (b) Show that if $\{a_n\}$ and $\{b_n\}$ are asymptotically equivalent then their relative error tends to zero. (3 mks)

   (c) Let a sequence $a_n = n$ and $b_n = 2n$. Show the relationship between a_n and b_n (3 mks)

   (d) If $X_1, X_2, ..., X_n$ is a sequence of random variables and if mean μ_n and standard deviation σ_n of X_n exists for all n and $\sigma_n \to -\infty$ as $n \to \infty$, then $x_n - \mu_n \to 0$ as $n \to \infty$ (3 mks)

   (e) Let $X_1, X_2, ..., X_n$ be observations that are i.i.d. Show that the sample mean is a consistent estimator for the population mean (5 mks)

   (f) i. Define asymptotic equivalence (2 mks)

   ii. Show that if

$$a_n = \begin{cases} \frac{1}{\sqrt{n}}, n = 1, 4, 9, ... \\ \frac{1}{n}, otherwise \end{cases}$$

   then $a_n \to 0$ but in irregular behavior. (3 mks)

   (g) Let $a_n = \frac{1}{n} + \frac{2}{n^2} + \frac{3}{n^3}$ and $b_n = \frac{1}{n}$. Check if a_n and b_n are equivalent (3 mks)

   (h) State the conditions necessary for the existence of the weak law of large numbers (4 mks)

QUESTION TWO (20 MARKS)

2. (a) Discuss any two uses of the large sample theory (4 mks)

   (b) State the conditions for $X_n = O_p(1)$ to hold (6 mks)

   (c) Show that if $F_n \to F$, then the convergence is uniform;

$$\sup_x |F_n| - F(x) \to 0$$

   as $n \to \infty$ (10 mks)

QUESTION THREE (20 MARKS)

3. (a) For a Geometric distribution $P(x) = 2^{-x}$; such that $x = 1, 2, \ldots$ prove that Chebyshev's inequality gives $P[|x - 2| \leq 2] > \frac{1}{2}$ while the actual probability is $\frac{15}{16}$ (10 mks)

   (b) Use the Chebyshev inequality to determine how many times a balanced coin must be tossed in order that the probability will be at least 0.80 that the ratio of the observed number of heads to the number of tosses will be between 0.4 and 0.6 (10 mks)

QUESTION FOUR (20 MARKS)

4. (a) Let $X_1, X_2, \ldots, X_n$ be i.i.d observations from the uniform distribution, $f(x;\theta) = \frac{1}{\theta}$, where θ is unknown positive parameter. Show that the MLE of θ is $\hat{\theta} = X_n = Max$ (6 mks)

   (b) State and prove the Slutsky's Theorem (10 mks)

   (c) Let $a_n = n$ and $b_n = n^3$. What is the relationship when a_n and b_n gives their logs (4 mks)

QUESTION FIVE (20 MARKS)

5. (a) Suppose X is a random variable with mean μ and variance σ^2, then for any positive number K

$$P\{|X - \mu| \geq K\sigma\} \leq \frac{1}{K^2}$$

(10 mks)

   (b) Suppose $X_n \to X$, show that for any continuous function $g(.)$, $g(X_n) \to g(X)$. (3 mks)

   (c) A dice is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the dice cannot be regarded as an unbiased one and find the limits between which the probability of a throw of 3 or 4 lies. (7 mks)