UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST-YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN STATISTICS

COURSE CODE: **STA 803**

COURSE TITLE: THEORY OF ESTIMATION

DATE: 16/08/2023 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 2 Hours

QUESTION ONE (20 MARKS)

a) Let X_1 *and* X_2 be independent Poisson variables with common expectations λ so that the joint distribution is

$$p(X_1 = x_1, X_2 = x_2) = \frac{\lambda^{x_1+x_2}}{x_1!\,x_2!}e^{-2\lambda}$$

Show that T is sufficient for λ

b) Consider the normal density function with parameter (μ, δ)

$f(x/\mu, \delta) = \frac{1}{2\pi\delta^2}\exp - \frac{1}{2\delta^2}(x-\mu)^2$ find sufficient statistics for μ, δ

QUESTION TWO (20 MARKS)

a) Define and state the significance of statistical inference

b) With illustrations discuss four properties of a good estimator

c) State the Rao – Blackwell Theorem

d) State and prove the Cramer Rao lower bound

QUESTION THREE (20 MARKS)

a) Highlight the properties of an exponential family and determine if normal density with unknown mean and variance 1 belongs to an exponential family

b) Consider $f(x; \theta) = \theta^x(1-\theta)^{1-x}$, determine if the density function belongs to an exponential family

c) When do we say that an exponential family is canonical?

QUESTION FOUR (20 MARKS)

Let $x_1, x_2, \ldots x_2$ be a random sample from a binomial experiment with parameters; n *and* p. Find;

a) The estimator of p

b) The asymptotic variance of the estimator

c) Bias of the estimator

d) Mean square error of the estimator

QUESTION FIVE (20 MARKS)

a) Describe step by step how the moment generating function of an exponential family can be obtained

a) Let $x_i; i = 1,2,...,n$ be a random sample from a normal distribution with unknown mean μ and known variance δ^2. Obtain the CR (Cramer Rao) lower bound for the unbiased estimator μ. Hence UMVU of μ

b) Show that for a random sample from a normal population, the sample variance S^2 is a consistent estimator of δ^2