

# **UNIVERSITY EXAMINATIONS 2017/2018 ACADEMIC YEAR** THIRD YEAR SECOND SEMESTER SPECIAL/ SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION AND

# **BACHELOR OF SCIENCE MATHEMATICS**

COURSE CODE: MAT 304

COURSE TITLE: COMPLEX ANALYSIS I

**DATE:** 12/10/18

**TIME**: 8 AM -10 AM

#### **INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

#### QUESTION ONE: COMPULSORY (30 MARKS)

- $f(z) = \sin z$ , find the Maclaurious series (5 marks)
- b) If  $z_1$  and  $z_2$  are complex numbers, prove that  $|z_1 + z_2| \le |z_1| + |z_2|$ (6 marks)
- c) For which values of Z is the function continuous  $f(z) = \frac{z}{(z-i)(z+i)}$ (4marks)
- d)Giventhat  $w = f(z) = z^2$ , find the values of w that correspond to z = -3 + i5(4 marks)
- e) Evaluate  $\oint_c \frac{e^z}{(z+1)^2} dz$  where c is the circle |z-1|=3(7 marks)
- f) Determine the poles of the function  $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ (4 marks)

#### QUESTION TWO (20 MARKS)

a) Evaluate  $f(z) = \frac{1}{1-z}$  at a = 3 using Taylors' series

(6 marks)

b) State and prove the Residue Theorem

- (5marks)
- c) Find the residues of  $f(z) = \frac{z^2 2z}{(z+1)^2(z^2+4)}$  at all its poles in the finite plane and hence evaluate  $\oint f(z)dz$ (9marks)

# QUESTION THREE (20 MARKS)

a) Evaluate  $\int_{1+z}^{z+3} (z^2+z) dz$  along the line joining the points (1, -1) and (2, 3)

(6marks)

- b) Evaluate  $\oint_c \frac{2z-1}{z(z+1)(z-1)} dz$ , where c is the *circle* |z| = 2(7 marks)
- c) Evaluate the integral  $\int_0^{4+2i} \overline{Z} dz$  along the curve  $Z = t^2 + it$  (7 marks)

### **QUESTION FOUR (20 MARKS)**

- a) Find the first four terms of the Taylor series expansion of  $f(z) = \ln(1+z)$ about the point z = 0
- b) Using Cauchy's integral formula, evaluate  $\int \frac{2z^2+z}{z^2-1}dz$  where C is |z-1|=1(7 marks)
- c) If f(z) is analytic within and on simple closed curve C and if a is any point within C, show that  $f(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z-a} dz$ (6 marks)

## QUESTION FIVE(20 MARKS)

- a) Evaluate  $\oint_C \frac{3z^2+z}{z^2-1} dz$  where C is a circle |z-1|=1 (10 marks) b) Locate and name the singularities in the finite Z-plane  $f(z)=\frac{z}{(z^2+4)^2}$ determine whether it is isolated singularity or not . (10 marks)