UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 422**

COURSE TITLE: **PDE II**

DATE: 12/10/18 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTIONONE (30 Mks)

(a) Solve the equation $S = 2x + 2y$ (15Mks)

(b) Find the solution of the wave equation

$$\frac{d^2u}{dt^2} - \frac{c^2d^2u}{dx^2} = 0$$

Satisfying the initial conditions when $u(x,0) = 0, u_t(x,0) = \sin 2x$ (15Mks)

QUESTION TWO (20Mks)

(a) Distinguish between

(i) linear and homogeneous differential equation. (2Mks)

(ii) linear and non-homogeneous differential equation (2Mks)

(iii) linearly dependant and linearly independent functions (2Mks)

(b) (i) define a wronskian (2Mks)

(ii) given that $f(x) = x^2, f_2(x) = \sin x \cos x$. Find $W(f_1, f_2)$ at $x = \frac{\pi}{4}$ (4Mks)

(c) Show that the equation $x^3y''' - 6xy' + 12y = 0$ has 3 linearly independent solutions of the form $y = x'$ (8Mks)

(d) Find the general solution of the equation $Xu_X - Yu_Y + u = X$ (10Mks)

QUESTION THREE (20 Mks)

(a) The general form of a linear's 1st order of p.d.e is

$$a(x,y)u_x + b(xy)u_y + c(xy) = d(xy)$$

Where $u_x = {du}/{dx}$ $u_y = {du}/{dy}$, and the coefficients a,b,c and d are functions of x and y in some domain D in xy plane. Show that the characteristic equation of the above p.d.e is given ${dy}/{dx} = {b}/{a}$ (14Mks)

(b) Find a particular integral of the equation $\frac{\partial^2 z}{\partial x^x} - \frac{\partial z}{\partial y} = e^{2x+y}$ (6Mks)

QUESTION FOUR (20 MKS)

A metal bar of length π has its ends isolated. If the initial temperature of the bar is

$x(\pi - x)$. Find the distribution of the temperature in the bar at a later time for the heat equation $K^2 u_{xx} = u_t$ satisfying

$$\left.\begin{matrix} u(0,t) = 0 \\ u(\pi,t) = 0 \end{matrix}\right\} 0 \leq + \leq \infty$$

$u(x, 0) = x(\pi x)$ (20Mks)

QUESTION FIVE (20MKS)

(a) Find a surface satisfying the differential equation $t = 6x^3 y$ which contains the two lines $y = o = z$ and $y = 1 = z$ (10Mks)

(b) Distinguish between Dirichlet's and Neumann conditions of heat condution

(6Mks)

(c) Solve the p.d.e d^2y/dx^2dy=2 (4Mks)