UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST YEAR FIRSTSEMESTER

MAIN EXAMINATION

FOR THE DEGREE MASTER OF SCIENCE IN APPLIED MATHEMATICS

COURSE CODE: **MAT 869/ 817**

COURSE TITLE: **COMPLEX ANALYSIS I**

DATE: 08/08/2023 **TIME: 8:00AM-11:00AM**

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

This Paper Consists of 2 Printed Pages. Please Turn Over.

QUESTION ONE (20 MARKS)

a) Determine the number of zeros of $2z^5 - 6z^2 + z + 1$ in $1 < |2| < 2$ (7mks)

b) Show that $cot^{-1}(z) = \frac{1}{2i} In \left(\frac{z+i}{z-i}\right)$ (5mks)

c) Use the Cauchy integral formula to evaluate $\oint_c \frac{2z^2+z}{z^2-1} dz$ where c is in the circle $|z-1| = 1$ (5mks)

d) Prove that $cosh^2(z) - sinh^2(z) = 1$ (3mks)

QUESTION TWO (20MARKS)

a) Determine the LFT that maps $z = 0, -i, -1$ to $w = i, 1, 0$ respectively (10mks)

b) Consider the triangle $A(0,1)$, $B(1,1)$ and $C(1,0)$. Find the image of ABC under $T(z) = z^2$. Discuss conformity of T at $B(1,1)$ and $C(1,0)$. (10mks)

QUESTION THREE (20MARKS)

a) Evaluate $\int_c (x^2 - iy^2)dz$ along

i. The parabola $y = 2x^2$ from $(1,1)$ *to* $(2,8)$ (5mks)

ii. The straight lines from $(1,1)$ *to* $(1,8)$ and $(1,8)$ *to* $(2,8)$ (5mks)

b) Prove that (10mks)

i. $\oint_c dz = 0$

ii. $\oint_c z dz = 0$

iii. $\oint_c (z - z_0)dz = 0$

QUESTION FOUR (20MARKS)

a) Find the residues of $f(z) = \frac{z^2+2z}{(z+1)^2(z^2+4)}$ at all its poles and hence evaluate $\oint_c f(z)dz$ (10mks)

b) Find the first four terms of a Tylor's series expansion of the function $f(z) = \frac{1}{(z-1)(z-3)}$ about the point $z = 4$ (10mks)

QUESTION FIVE (20MARKS)

a) Prove that the function $f_1(z) = \int_0^\infty t^3 e^{-zt} dt$ is analytic at all points z for which $R_e\ z > 0$. Find an analytic continuation of $f_1(z)$ into the left plane $R_e\ z < 1$(10mks)

b) Prove the theorem, if $f(z) = U + iV$ is analytic (differential) and $f^1(z) = 0$ in a region R then the mapping $w = f(z)$ is conformal at all points in R. (10mks)